$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \end{pmatrix}$$

neu $IV = 1 \cdot IV - 1 \cdot I$
neu $VII = 1 \cdot VII - 1 \cdot I$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & (-1) & (-1) & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & (-1) & (-1) & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 0 \\ 15 \\ 15 \\ 0 \\ 15 \end{pmatrix}$$

tausche: $IV <-> II$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (-1) & (-1) & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & (-1) & (-1) & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 0 \\ 15 \\ 15 \\ 15 \\ 15 \\ 0 \\ 15 \end{pmatrix}$$

neu $V <-> II$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & (-1) & (-1) & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & (-1) & (-1) & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 0 \\ 15 \\ 0 \\ 15 \end{pmatrix}$$

neu $V = 1 \cdot V - (-1) \cdot II$
neu $VII = 1 \cdot VII - (-1) \cdot II$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & (-1) & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & (-1) & 0 & 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \end{pmatrix}$$

tausche: $V <-> III$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & (-1) & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & (-1) & 0 & 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \end{pmatrix}$$

neu $VI <-> III$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & (-1) & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & (-1) & 0 & 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \end{pmatrix}$$

neu $VIII <-> III$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & (-1) & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & (-1) & 0 & 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \end{pmatrix}$$

neu $VI = 1 \cdot VI - (-1) \cdot III$
neu $VII = 1 \cdot VII - (-1) \cdot III$
neu $VIII = 1 \cdot VIII - 1 \cdot III$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & (-1) & 1 & (-1) & 0 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 30 \\ 30 \\ 0 \end{pmatrix}$$

neu $VI = 1 \cdot VI - 1 \cdot IV$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & (-1) & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & (-1) & 1 & (-1) & 0 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 30 \\ 0 \end{pmatrix}$$

tausche: $VI <-> V$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & (-1) & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 3 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & (-1) & 1 & (-1) & 0 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 30 \\ 0 \end{pmatrix}$$

neu $VII = 1 \cdot VII - 3 \cdot V$
neu $VIII = 1 \cdot VIII - (-1) \cdot V$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & (-1) & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 & (-5) & (-2) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ (-15) \\ 15 \end{pmatrix}$$

tausche: $VII <-> VI$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & (-1) & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & (-5) & (-2) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ (-15) \\ 15 \\ 15 \end{pmatrix}$$

neu $VIII = 1 \cdot VIII - 1 \cdot VII$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & (-1) & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & (-5) & (-2) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ (-15) \\ 15 \\ 0 \end{pmatrix}$$

Die Matrix wird mit Nullzeilen zu einer (9×9)-Matrix ergänzt

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & (-1) & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & (-5) & (-2) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 15 \\ 15 \\ 15 \\ (-15) \\ 15 \\ 0 \\ 0 \end{pmatrix}$$

neu $VI = 1 \cdot VI - (-5) \cdot VII$
neu $V = 1 \cdot V - 2 \cdot VII$
neu $III = 1 \cdot III - 1 \cdot VII$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & (-1) & (-1) \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & (-1) & 0 & (-1) & (-2) \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 0 \\ 15 \\ (-15) \\ 60 \\ 15 \\ 0 \\ 0 \end{pmatrix}$$

neu $V = 3 \cdot V - (-1) \cdot VI$
neu $IV = 3 \cdot IV - 1 \cdot VI$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & (-1) & (-1) \\ 0 & 0 & 0 & 3 & 3 & 0 & 0 & (-3) & (-6) \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 0 \\ (-15) \\ 15 \\ 60 \\ 15 \\ 0 \\ 0 \end{pmatrix}$$

neu $IV = 1 \cdot IV - 1 \cdot V$
neu $III = 3 \cdot III - 1 \cdot V$
neu $II = 3 \cdot II - 1 \cdot V$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & (-3) & (-3) \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & (-3) & (-6) \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 15 \\ 30 \\ (-15) \\ (-30) \\ 15 \\ 60 \\ 15 \\ 0 \\ 0 \end{pmatrix}$$

neu $I = 3 \cdot I - 1 \cdot III$

$$\begin{pmatrix} 3 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & (-3) & (-3) \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & (-3) & (-6) \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 60 \\ 30 \\ (-15) \\ (-30) \\ 15 \\ 60 \\ 15 \\ 0 \\ 0 \end{pmatrix}$$

neu $I = 1 \cdot I - 1 \cdot II$

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & (-3) & (-3) \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & (-3) & (-6) \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 30 \\ 30 \\ (-15) \\ (-30) \\ 15 \\ 60 \\ 15 \\ 0 \\ 0 \end{pmatrix}$$

neu $I = I/3$
neu $II = II/3$
neu $III = III/3$
neu $IV = IV/3$
neu $V = V/3$
neu $VI = VI/3$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \vec{x} = \begin{pmatrix} 10 \\ 10 \\ -5 \\ -10 \\ 5 \\ 20 \\ 15 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} 10 \\ 10 \\ -5 \\ -10 \\ 5 \\ 20 \\ 15 \\ 0 \\ 0 \end{pmatrix} + r \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 0 \\ -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \\ 0 \\ -2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$